

How to Scale Up the Spectral Efficiency of Multi-way Massive MIMO Relaying?

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Abstract—This paper considers a decode-and-forward (DF) multi-way massive multiple-input multiple-output (MIMO) relay system where many users exchange their data with the aid of a relay station equipped with a massive antenna array. We propose a new transmission protocol which leverages successive cancellation decoding and zero-forcing (ZF) at the users. By using properties of massive MIMO, a tight analytical approximation of the spectral efficiency is derived. We show that our proposed scheme uses only half of the time-slots required in the conventional scheme (in which the number of time-slots is equal to the number of users [1]), to exchange data across different users. As a result, the sum spectral efficiency of our proposed scheme is nearly double the one of the conventional scheme, thereby boosting the performance of multi-way massive MIMO to unprecedented levels.

Index Terms—Amplify-and-forward, decode-and-forward, maximum-ratio processing, multi-way relay massive MIMO.

I. INTRODUCTION

In the past few years, massive MIMO technology has attracted significant research attention for its ability to improve the spectral and energy efficiency [2], [3]. In massive MIMO systems, many users can be served by a base station equipped with very large antenna arrays. With very large antenna arrays at the base station, the channels between different users become pairwise orthogonal, and hence, the noise and inter-user interference reduce noticeably without improving the complexity of the system [3]. Furthermore, by using time division duplex (TDD) mode, the channel estimation overhead depends only on the number of active users regardless of number of base station antennas [4]. This makes massive MIMO scalable and become one of the key candidates for future wireless communication systems.

On a parallel avenue, multi-way relaying networks have also been investigated to enhance the robustness against the channel variations in distinguished areas, where the direct channels among users are unavailable due to large obstacle and/or heavy path loss in the propagation environment [5]. With the help of the relay station, users that are geographically separated can communicate or exchange their data-bearing symbols much easier. Moreover, a significant number of papers demonstrate that multi-way relaying networks provide much higher spectral efficiency and communication reliability compared to one-way or two-way relaying systems [6], [7].

The combination of multi-way relaying and massive MIMO is very promising since it reaps all benefits of both technolo-

gies. Recently, some papers have evaluated the performance of multi-way relaying networks with massive arrays at the relay [8], [9]. In these works, the authors showed that multi-way massive MIMO relay systems can offer huge spectral and energy efficiency. In addition, by using simple linear processing (e.g. ZF and maximum ratio processing) and employing a large number of antennas at the relay station, the transmit power of each user can be scaled down proportionally to the number of relay antennas, while maintaining a given quality of service. However, all of aforementioned studies considered a conventional transmission protocol which requires K time-slots to exchange data among K users.

Different with previous works, in this paper we propose a novel transmission protocol for multi-way massive MIMO relay networks which requires only $\lceil \frac{K-1}{2} \rceil + 1$ time-slots for the information exchange among the K users. We consider the DF operation at the relay, and assume that the relay and the users have perfect knowledge of the channel state information (CSI). We derive an approximate closed-form expression for the spectral efficiency. The approximation is shown to be very tight, especially when the number of relay antennas is large.

Notations: Matrices and vectors are expressed as upper and lower case boldface letter, respectively. The superscripts $(\cdot)^H$ and $\text{Tr}(\cdot)$ stand for Hermitian transpose and the trace, respectively. We denote by \mathbf{a}_k the k -th column of matrix \mathbf{A} . The symbol $\|\cdot\|$ indicates the norm of a vector. The notation $\mathbb{E}\{\cdot\}$ is the expectation operator. The notation $[\mathbf{A}]_{mn}$ or a_{mn} denotes the (m, n) -th element of matrix \mathbf{A} , and \mathbf{I}_K is the $K \times K$ identity matrix.

II. SYSTEM MODEL

We consider a DF multi-way relay networks with a very large antenna array at the relay station. The system includes one relay station equipped with M antennas and K single-antenna users. The bearing-messages from K users are exchanged with the help of the relay station. Each user wants to detect the signals transmitted from $K - 1$ other users. We assume that the users and the relay station operate in half-duplex mode and know perfectly CSI. Furthermore, we assume that the direct links (user-to-user links) are unavailable due to large path loss and/or severe shadowing.

The channel matrix between the K users and M antennas at the relay is denoted by $\mathbf{G} \in \mathbb{C}^{M \times K}$ and is modeled as

$$\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2}, \quad (1)$$

where $\mathbf{H} \in \mathbb{C}^{M \times K}$ models small-scale fading with independent $\mathcal{CN}(0, 1)$ components, and $\mathbf{D} \in \mathbb{C}^{K \times K}$ is the diagonal matrix of large-scale fading (path loss and log-normal attenuation). Let g_{mk} and h_{mk} be the (m, k) -th element of \mathbf{G} and \mathbf{H} , respectively. Then

$$g_{mk} = \sqrt{\beta_k} h_{mk}, \quad (2)$$

where β_k is the k -th diagonal element of \mathbf{D} . In general, the transmission protocol is divided into two phases: multiple-access phase and broadcast phase. In the multiple-access phase, all K users transmit signals to the relay station. In the broadcast phase, the relay station broadcast signals (which are decoded in the multi-access phase) to the users. In the next sections, we will first present the conventional transmission protocol, followed by the proposed transmission scheme.

III. CONVENTIONAL TRANSMISSION PROTOCOL

In this section, we first summarize a conventional transmission protocol tailored to multi-way massive DF relaying networks. The uplink and downlink spectral efficiencies are then provided in closed-form.

A. Multiple-Access Phase

This phase requires only one time-slot. All the K users transmit their data to the relay in the same time-frequency resource. The $M \times 1$ received vector at the relay is

$$\mathbf{y}_R = \sqrt{P_u} \mathbf{G} \mathbf{x} + \mathbf{n}_R, \quad (3)$$

where $\mathbf{x} \triangleq [x_1, x_2, \dots, x_K]^T$ is the signal vector transmitted from the K users, with $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_K$, \mathbf{n}_R is the noise vector with i.i.d. $\mathcal{CN}(0, 1)$ elements, and P_u is the normalized transmit power of each user.

After receiving the transmitted signals from the K users, the relay employs maximum ratio combining scheme by multiplying \mathbf{y}_R with \mathbf{G}^H as follows:

$$\mathbf{r} = \mathbf{G}^H \mathbf{y}_R = \sqrt{P_u} \mathbf{G}^H \mathbf{G} \mathbf{x} + \mathbf{G}^H \mathbf{n}_R. \quad (4)$$

Then, the k -th element of \mathbf{r} , denoted by r_k , is used to decode the signal transmitted from user k . From (4), r_k is given by

$$r_k = \sqrt{P_u} \|\mathbf{g}_k\|^2 x_k + \sqrt{P_u} \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{g}_k^H \mathbf{g}_i x_i + \mathbf{g}_k^H \mathbf{n}_R, \quad (5)$$

where \mathbf{g}_k is the k -th column of \mathbf{G} . Therefore, the uplink spectral efficiency of the system in (5) (measured in bit/s/Hz) is given by

$$R_k^{\text{ul}} = \mathbb{E} \left\{ \log_2 \left(1 + \frac{P_u \|\mathbf{g}_k\|^4}{P_u \sum_{\substack{i=1 \\ i \neq k}}^K |\mathbf{g}_k^H \mathbf{g}_i|^2 + \|\mathbf{g}_k\|^2} \right) \right\}. \quad (6)$$

By using Jensen's inequality, a closed-form expression lower bound of the spectral efficiency (6) is given by [3, Eq. (16)]

$$R_k^{\text{ul}} \geq \tilde{R}_k^{\text{ul}} = \log_2 \left(1 + \frac{P_u (M-1) \beta_k}{P_u \sum_{\substack{i=1 \\ i \neq k}}^K \beta_i + 1} \right). \quad (7)$$

B. Broadcast Phase

In this phase, the relay station transmits all signals decoded in the multiple-access phase to all users in $K-1$ time slots. In the t time-slot, the relay aims to transmit $x_{j(k,t)}$ to user k , $k = 1, \dots, K$, where

$$j(k, t) \triangleq \begin{cases} (k+t) \text{ modulo } K, & \text{if } (k+t) \neq K \\ K, & \text{otherwise.} \end{cases} \quad (8)$$

More precisely, in the t -th time-slot, the relay station transmits

$$\mathbf{s}^{(t)} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_i x_{j(i,t)}, \quad (9)$$

where P_r is the normalized transmit power at the relay. Then, the received signal at the k -th user is

$$\begin{aligned} y_k^{(t)} &= \mathbf{g}_k^H \mathbf{s}^{(t)} + n_k^{(t)} \\ &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,t)} + n_k^{(t)}, \end{aligned} \quad (10)$$

respectively.

The k -th user knows its own transmitted signal x_k (or $x_{j(k-t,t)}$), so it can remove the self-interference prior to decoding. The received signal after self-interference cancellation is

$$\begin{aligned} \tilde{y}_k^{(t)} &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \|\mathbf{g}_k\|^2 x_{j(k,t)} \\ &+ \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,t) \neq j(k,t), j(k-t,t)}}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,t)} + n_k^{(t)}. \end{aligned} \quad (11)$$

The corresponding downlink spectral efficiency for the t -th time-slot is

$$R_k^{\text{dl},(t)} = \mathbb{E} \left\{ \log_2 \left(1 + \frac{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \sum_{\substack{i=1 \\ j(i,t) \neq j(k,t), j(k-t,t)}}^K |\mathbf{g}_k^H \mathbf{g}_i|^2 + 1}} \right) \right\}. \quad (12)$$

Proposition 1: The spectral efficiency $R_k^{\text{dl},(t)}$ given by (12) can be lower bounded by

$$\begin{aligned} R_k^{\text{dl},(t)} &\geq \tilde{R}_k^{\text{dl},(t)} \\ &= \log_2 \left(1 + \frac{P_r (M-1) (M-2) \beta_k^2}{P_r (M-2) \beta_k \sum_{\substack{i=1 \\ j(i,t) \neq j(k,t), j(k-t,t)}}^K \beta_i + M \sum_{i=1}^K \beta_i} \right). \end{aligned} \quad (13)$$

Proof: See Appendix VII-A. ■

IV. MULTI-WAY TRANSMISSION WITH SUCCESSIVE CANCELATION DECODING

In this section, we propose a novel transmission scheme which requires only $\lceil \frac{K-1}{2} \rceil + 1$ time-slots for the information exchange among the K users.

A. Multiple-Access Phase

The multiple-access phase is the same as the one of conventional transmission scheme. See Section III-A.

B. Broadcast Phase

Here, we need only $\lceil \frac{K-1}{2} \rceil$ time-slots to transmit all K symbols to all users. The main idea is that: at a given time-slot, the k -th user subtracts all symbols decoded in previous time-slots prior to decoding the desired symbol. Furthermore, after $\lceil \frac{K-1}{2} \rceil$ time-slots, user k receives $\lceil \frac{K-1}{2} \rceil$ signals, and each signal is a linear combination of $K - \lceil \frac{K-1}{2} \rceil - 1$ symbols. So it can detect all $K - \lceil \frac{K-1}{2} \rceil - 1$ symbols without any inter-user interference through the zero-forcing technique. A detailed presentation of the proposed scheme is now provided.

1) First time-slot: The relay intends to send $x_{j(k,1)}$ to the k -th user, for $k = 1, \dots, K$. The signal vector transmitted from the relay is

$$\mathbf{s}^{(1)} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_i x_{j(i,1)}. \quad (14)$$

Thus, the received signal at the k -th user is

$$\begin{aligned} y_k^{(1)} &= \mathbf{g}_k^H \mathbf{s}^{(1)} + n_k^{(1)} \\ &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,1)} + n_k^{(1)}, \end{aligned} \quad (15)$$

where $n_k^{(1)} \sim \mathcal{CN}(0, 1)$ is the additive noise at the k -th user in the first time-slot. Since user k knows its transmitted signal x_k (or $x_{j(k-1,1)}$), it can subtract the self-interference before detecting signal $x_{j(k,1)}$. Therefore, the received signal at user k after self-interference cancellation is

$$\begin{aligned} \tilde{y}_k^{(1)} &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \|\mathbf{g}_k\|^2 x_{j(k,1)} \\ &+ \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,1) \notin \mathcal{V}_{k,1}}}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,1)} + n_k^{(1)}, \end{aligned} \quad (16)$$

where

$$\mathcal{V}_{k,t} \triangleq \{j(k-t, t), j(k-t+1, t), \dots, j(k, t)\}. \quad (17)$$

Then, the corresponding spectral efficiency is given by

$$\begin{aligned} R_k^{\text{dl},(1)} &= \mathbb{E} \left\{ \log_2 \left(1 + \frac{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \sum_{\substack{i=1 \\ j(i,1) \notin \mathcal{V}_{k,1}}}^K |\mathbf{g}_k^H \mathbf{g}_i|^2 + 1}} \right) \right\}. \end{aligned} \quad (18)$$

2) Second time-slot: The relay intends to send $x_{j(k,2)}$ to the k -th user, for $k = 1, \dots, K$. The signal vector transmitted from the relay is

$$\mathbf{s}^{(2)} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_i x_{j(i,2)}, \quad (19)$$

and hence, the signal received at the k -th user is

$$\begin{aligned} y_k^{(2)} &= \mathbf{g}_k^H \mathbf{s}^{(2)} + n_k^{(2)} \\ &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,2)} + n_k^{(2)}. \end{aligned} \quad (20)$$

The k -th user knows its own transmitted symbol x_k as well as the symbol detected in the first time-slot $x_{j(k,1)}$, so it can subtract these symbols before detecting the desired signal $x_{j(k,2)}$. The received signal at the k -th user after subtracting the above symbols is

$$\begin{aligned} \tilde{y}_k^{(2)} &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \|\mathbf{g}_k\|^2 x_{j(k,2)} \\ &+ \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,2) \notin \mathcal{V}_{k,2}}}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,2)} + n_k^{(2)}. \end{aligned} \quad (21)$$

Then, the spectral efficiency of user k at the second time-slot is

$$\begin{aligned} R_k^{\text{dl},(2)} &= \mathbb{E} \left\{ \log_2 \left(1 + \frac{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \sum_{\substack{i=1 \\ j(i,2) \notin \mathcal{V}_{k,2}}}^K |\mathbf{g}_k^H \mathbf{g}_i|^2 + 1}} \right) \right\}. \end{aligned} \quad (22)$$

3) t -th time-slot: At the t -time-slot, the relay intends to send $x_{j(k,t)}$ to the k -th user, for $k = 1, \dots, K$. The signal vector transmitted from the relay is

$$\mathbf{s}^{(t)} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_i x_{j(i,t)}. \quad (23)$$

Then, the k -th user sees

$$\begin{aligned} y_k^{(t)} &= \mathbf{g}_k^H \mathbf{s}^{(t)} + n_k^{(t)} \\ &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{i=1}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,t)} + n_k^{(t)}. \end{aligned} \quad (24)$$

The k -th users know its own transmitted symbols x_k . Furthermore, it also knows its detected symbols in previous time-slots. So it knows $\{x_{j(k-1,1)}, x_{j(k,1)}, x_{j(k,2)}, \dots, x_{j(k,t-1)}\}$, and, hence, it can remove these symbols to obtain

$$\begin{aligned} \tilde{y}_k^{(t)} &= \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \|\mathbf{g}_k\|^2 x_{j(k,t)} \\ &+ \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,t) \notin \mathcal{V}_{k,t}}}^K \mathbf{g}_k^H \mathbf{g}_i x_{j(i,t)} + n_k^{(t)}. \end{aligned} \quad (25)$$

Then, the spectral efficiency of the k -th user at the t -th time-slot is

$$\begin{aligned} R_k^{\text{dl},(t)} &= \mathbb{E} \left\{ \log_2 \left(1 + \frac{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \sum_{\substack{i=1 \\ j(i,t) \notin \mathcal{V}_{k,t}}}^K |\mathbf{g}_k^H \mathbf{g}_i|^2 + 1}} \right) \right\}. \end{aligned} \quad (26)$$

Proposition 2: The spectral efficiency $R_k^{\text{dl},(t)}$ given by (26) can be lower bounded by

$$\begin{aligned} R_k^{\text{dl},(t)} &\geq \tilde{R}_k^{\text{dl},(t)} \\ &= \log_2 \left(1 + \frac{P_r(M-1)(M-2)\beta_k^2}{P_r(M-2)\beta_k \sum_{\substack{i=1 \\ j(i,t) \notin \mathcal{V}_{k,t}}}^K \beta_i + M \sum_{i=1}^K \beta_i} \right). \end{aligned} \quad (27)$$

Proof: Following a similar methodology as the proof of Proposition 1. ■

4) After $t' = \lceil \frac{K-1}{2} \rceil$ time-slots, the k -th user has received t' signals (the t -th received signal is given by (24)). Furthermore, it has decoded t' symbols. So it can subtract all t' detected symbols from each received signal to obtain the following results:

$$\begin{cases} \tilde{y}_{k,1}^{(t')} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,t') \notin \mathcal{V}_{k,t'}}}^K \mathbf{g}_k^H \mathbf{g}_{j(k,1)} x_{j(i,t')} + n_{k,1}^{(t')}, \\ \tilde{y}_{k,2}^{(t')} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,t') \notin \mathcal{V}_{k,t'}}}^K \mathbf{g}_k^H \mathbf{g}_{j(k,2)} x_{j(i,t')} + n_{k,2}^{(t')}, \\ \vdots \\ \tilde{y}_{k,t'}^{(t')} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \sum_{\substack{i=1 \\ j(i,t') \notin \mathcal{V}_{k,t'}}}^K \mathbf{g}_k^H \mathbf{g}_{j(k,t')} x_{j(i,t')} + n_{k,t'}^{(t')}. \end{cases} \quad (28)$$

We can see that we have t' equations, each equation has $(K - t' - 1)$ unknown variables $\{x_{j(i,t')}\}$. Since $t' = \lceil \frac{K-1}{2} \rceil$, the number of equations is greater than or equal to the number of unknown variables. Therefore, the k -th user can detect all

remaining $(K - t' - 1)$ symbols $\{x_{j(i,t')}\}$ via the ZF scheme as follows. Denote by

$$\bar{\mathbf{y}}_k^{(t')} \triangleq \begin{bmatrix} \tilde{y}_{k,1}^{(t')} \\ \tilde{y}_{k,2}^{(t')} \\ \vdots \\ \tilde{y}_{k,t'}^{(t')} \end{bmatrix}, \quad \bar{\mathbf{n}}_k^{(t')} \triangleq \begin{bmatrix} n_{k,1}^{(t')} \\ n_{k,2}^{(t')} \\ \vdots \\ n_{k,t'}^{(t')} \end{bmatrix}, \quad (29)$$

$$\mathbf{A}_k \triangleq \begin{bmatrix} \mathbf{g}_k^H \mathbf{g}_{j(k,1)} & \mathbf{g}_k^H \mathbf{g}_{j(k,2)} & \cdots & \mathbf{g}_k^H \mathbf{g}_{j(k,K-t'-1)} \\ \mathbf{g}_k^H \mathbf{g}_{j(k,2)} & \mathbf{g}_k^H \mathbf{g}_{j(k,3)} & \cdots & \mathbf{g}_k^H \mathbf{g}_{j(k,K-t')} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{g}_k^H \mathbf{g}_{j(k,t')} & \mathbf{g}_k^H \mathbf{g}_{j(k,t'+1)} & \cdots & \mathbf{g}_k^H \mathbf{g}_{j(k,K-2)} \end{bmatrix}, \quad (30)$$

and

$$\bar{\mathbf{x}} \triangleq [x_{j(k,t'+1)} \quad x_{j(k,t'+2)} \quad \cdots \quad x_{j(k,K-1)}]^T. \quad (31)$$

Then, (28) can be rewritten in matrix-vector form as

$$\bar{\mathbf{y}}_k^{(t')} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \mathbf{A}_k \bar{\mathbf{x}} + \bar{\mathbf{n}}_k^{(t')}. \quad (32)$$

The k -th user applies the ZF scheme to decode the remaining symbols as follows:

$$\tilde{\mathbf{r}}_k^{(t')} = \mathbf{Z}^T \bar{\mathbf{y}}_k^{(t')} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} \mathbf{Z}^T \mathbf{A}_k \bar{\mathbf{x}} + \mathbf{Z}^T \bar{\mathbf{n}}_k^{(t')}, \quad (33)$$

where

$$\mathbf{Z}^T \triangleq (\mathbf{A}_k^H \mathbf{A}_k)^{-1} \mathbf{A}_k^H. \quad (34)$$

The n -th element of $\tilde{\mathbf{r}}_k^{(t')}$ will be used to detect $x_{j(k,t'+n)}$. From (33) and the fact that $\mathbf{Z}^T \mathbf{A}_k = \mathbf{I}_{K-(t'+1)}$, the n -th element of $\tilde{\mathbf{r}}_k^{(t')}$ is given by

$$\tilde{r}_{k,n}^{(t')} = \sqrt{\frac{P_r}{M \sum_{i=1}^K \beta_i}} x_{j(k,t'+n)} + \mathbf{z}_n^T \bar{\mathbf{n}}_k^{(t')}. \quad (35)$$

Thus, the corresponding spectral efficiency of the system in (35) is

$$\begin{aligned} R_k^{\text{dl},(t'+n)} &= \mathbb{E} \left\{ \log_2 \left(1 + \frac{\frac{P_r}{M \sum_{i=1}^K \beta_i}}{\|\mathbf{z}_n\|^2} \right) \right\} \\ &= \mathbb{E} \left\{ \log_2 \left(1 + \frac{P_r}{M \sum_{i=1}^K \beta_i \left[(\mathbf{A}_k^H \mathbf{A}_k)^{-1} \right]_{nn}} \right) \right\}. \end{aligned} \quad (36)$$

Since (36) has a complicated form that involves a matrix inverse, we cannot obtain an exact closed-form. However, thanks to the trace lemma and law of large numbers (as M goes to infinity) [10], we can obtain the following approximating result.

Proposition 3: As $M \rightarrow \infty$, the spectral efficiency $R_k^{\text{dl},(t'+n)}$ given by (36) converges to

$$R_k^{\text{dl},(t'+n)} \rightarrow \log_2 \left(1 + \frac{P_r \beta_k \sum_{i=1}^{t'} \beta_{j(k,n+i-1)}}{\sum_{i=1}^K \beta_i} \right). \quad (37)$$

Proof: See Appendix VII-B. ■

V. NUMERICAL RESULTS

In this section, we provide numerical results to evaluate the performance of our proposed scheme. We consider the sum spectral efficiency, defined as

$$\text{SE}_{\text{sum}} = \frac{1}{t' + 1} \sum_{k=1}^K \sum_{t=1}^{K-1} \min(R_k^{\text{ul}}, R_k^{\text{dl},(t)}), \quad (38)$$

where $t' = \lceil \frac{K-1}{2} \rceil$ is the t' -th time-slot of the transmission protocol in broadcast phase.

First, we examine the tightness of our analytical results. Figure 1 shows the sum spectral efficiency of our proposed scheme versus the number of relay antennas with different K for the simple case $\beta_k = 1, \forall k$. The “analysis” curves represent our analytical results obtained by using the lower bounds (7), (27), and the asymptotic result (37). The “simulation” curves are generated from the outputs of a Monte-Carlo simulator using (6), (26), and (36). We can see that the proposed approximation is very tight, even with small number of antennas. Furthermore, as expected, the sum spectral efficiency increases significantly when the number of relay antennas increases.

We next compare the performance of our proposed scheme with the one of the conventional DF scheme (Section III-B) and the conventional AF scheme in [11] (see Fig. 2). We can see that our proposed scheme significantly outperforms other schemes. The sum spectral efficiency of our proposed scheme improves by factors of nearly 2 and 3 compared with the conventional DF scheme and the conventional AF scheme, respectively. This is due to the fact that with the conventional DF scheme, we need in total K time-slots to exchange the information among the K users, while with our proposed scheme, we need only $\lceil \frac{K-1}{2} \rceil + 1$.

Finally, we consider a more practical scenario where the large-scale fading β_k changes depending on the locations of users and the shadow fading. To generate the large-scale fading, we use the same model as in [1]. Figure 3 illustrates the cumulative distribution of the sum spectral efficiency of our proposed scheme for $K = 5, 7$, and 10. As expected, the sum spectral efficiency increases when K increases. The 95%-likely sum spectral efficiency with $K = 10$ is about 14.5 bit/s/Hz which is nearly 4 times and 2 times higher than that with $K = 5$ and $K = 7$, respectively.

VI. CONCLUSION

We proposed a novel and useful transmission scheme for multi-way massive MIMO relay systems with decode-and-forward protocol at the relay. While the conventional scheme needs K time-slots to exchange all data among K users, our proposed scheme, which is based on successive cancellation decoding, needs only $\lceil \frac{K-1}{2} \rceil + 1$ time-slots. Thus, the sum spectral efficiency of our proposed scheme is nearly double the sum spectral efficiency of the conventional scheme.

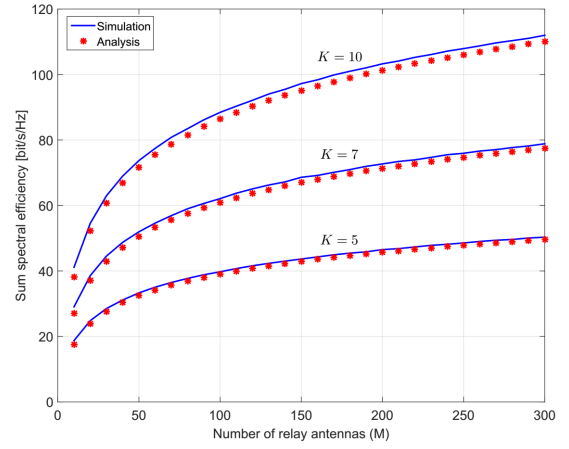


Fig. 1. The sum spectral efficiency of the system model with different K versus the number of relay antennas. We set $P_u = 0$ dB, $P_r = 10$ dB, $\beta_k = 1$.

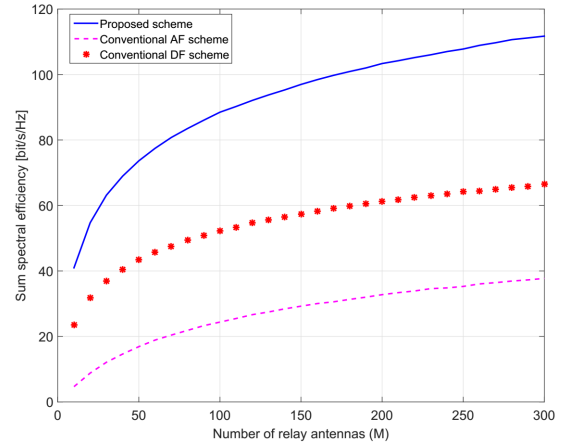


Fig. 2. The comparison of the sum spectral efficiency with different schemes versus the number of relay antennas. We choose $P_u = 0$ dB, $P_r = 10$ dB, $K = 10$, $\beta_k = 1$.

VII. APPENDICES

A. Proof of Proposition 1

By using Jensen’s inequality, we obtain

$$R_k^{\text{dl},(t)} \geq \tilde{R}_k^{\text{dl},(t)} = \log_2 \left(1 + \left(\mathbb{E} \left\{ \mathcal{X}_k^{(t)} \right\} \right)^{-1} \right), \quad (39)$$

where

$$\mathcal{X}_k^{(t)} \triangleq \frac{\frac{P_r}{M \sum_{i=1}^K \beta_i} \sum_{\substack{i=1 \\ j(i,t) \neq j(k,t), j(k-t,t)}}^K |\mathbf{g}_k^H \mathbf{g}_i|^2 + 1}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}. \quad (40)$$

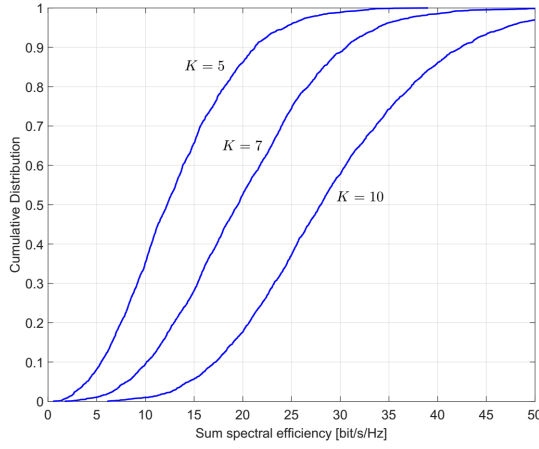


Fig. 3. Cumulative distribution of the sum spectral efficiency for different K . We choose $P_u = 0$ dB, $P_r = 10$ dB, $M = 100$.

By dividing the numerator and the denominator of the right-hand side of (40) by $\|\mathbf{g}_k\|^2$, we get

$$\mathbb{E}\left\{\mathcal{X}_k^{(t)}\right\} = \mathbb{E}\left\{\frac{\sum_{i=1}^K |\tilde{g}_i|^2}{\|\mathbf{g}_k\|^2} + \frac{1}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}\right\}, \quad (41)$$

where $\tilde{g}_i \triangleq \frac{\mathbf{g}_k^H \mathbf{g}_i}{\|\mathbf{g}_k\|^2}$. Conditioned on \mathbf{g}_k , \tilde{g}_i is Gaussian distributed with zero mean and variance β_i . Since the variance of \tilde{g}_i does not depend on \mathbf{g}_k , \tilde{g}_i is a $\mathcal{CN}(0, \beta_i)$ random variable and is independent of \mathbf{g}_k . Therefore,

$$\begin{aligned} \mathbb{E}\left\{\mathcal{X}_k^{(t)}\right\} &= \sum_{i=1}^K \mathbb{E}\left\{|\tilde{g}_i|^2\right\} \mathbb{E}\left\{\frac{1}{\|\mathbf{g}_k\|^2}\right\} \\ &\quad + \mathbb{E}\left\{\frac{1}{\frac{P_r}{M \sum_{i=1}^K \beta_i} \|\mathbf{g}_k\|^4}\right\}. \end{aligned} \quad (42)$$

By using [12, Lemma 2.10], we obtain $\mathbb{E}\left\{\frac{1}{\|\mathbf{g}_k\|^2}\right\} = \frac{1}{M-1}$ and $\mathbb{E}\left\{\frac{1}{\|\mathbf{g}_k\|^4}\right\} = \frac{M}{(M-1)^3 - (M-1)}$, and hence, we arrive at (13).

B. Proof of Proposition 3

From (30), the (m, n) -th element of $\mathbf{A}_k^H \mathbf{A}_k$ is given by

$$[\mathbf{A}_k^H \mathbf{A}_k]_{mn} = \sum_{i=1}^{t'} \mathbf{g}_k^H \mathbf{g}_{j(k, i-1+m)} \mathbf{g}_{j(k, i-1+n)}^H \mathbf{g}_k. \quad (43)$$

Using the trace lemma [10, Lemmas 4, 5], we have

$$\frac{1}{M} \mathbf{g}_k^H \mathbf{g}_{j(k, i)} \mathbf{g}_{j(k, i)}^H \mathbf{g}_k - \frac{\beta_k}{M} \text{Tr}\left(\mathbf{g}_{j(k, i)} \mathbf{g}_{j(k, i)}^H\right) \xrightarrow[M \rightarrow \infty]{a.s.} 0, \quad (44)$$

where “a.s.” stands for almost sure convergence.

Since $\text{Tr}\left(\mathbf{g}_{j(k, i)} \mathbf{g}_{j(k, i)}^H\right) = \|\mathbf{g}_{j(k, i)}\|^2$, and from the law of large numbers, we get

$$\frac{1}{M} \text{Tr}\left(\mathbf{g}_{j(k, i)} \mathbf{g}_{j(k, i)}^H\right) \xrightarrow[M \rightarrow \infty]{a.s.} \beta_{j(k, i)}. \quad (45)$$

The substitution of (45) into (44) yields

$$\frac{1}{M} \mathbf{g}_k^H \mathbf{g}_{j(k, i)} \mathbf{g}_{j(k, i)}^H \mathbf{g}_k \xrightarrow[M \rightarrow \infty]{a.s.} \beta_k \beta_{j(k, i)}. \quad (46)$$

Similarly, we obtain

$$\frac{1}{M} \mathbf{g}_{j(k, i)}^H \mathbf{g}_k \mathbf{g}_k^H \mathbf{g}_{j(k, i+1)} \xrightarrow[M \rightarrow \infty]{a.s.} 0. \quad (47)$$

From (43), (46), and (47), we have

$$M \left[(\mathbf{A}_k^H \mathbf{A}_k)^{-1} \right]_{nn} \xrightarrow[M \rightarrow \infty]{a.s.} \frac{1}{\beta_k \sum_{i=1}^{t'} \beta_{j(k, n+i-1)}}. \quad (48)$$

Substituting (48) into (36), we obtain (37).

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